Mathematics in Engineering*

By SIDNEY FRANKEL

Federal Telecommunication Laboratories, Incorporated, Nutley, New Jersey

AS FAR AS the engineer is concerned, mathematics is merely a tool to help him organize his thinking.

When man manipulates and attempts to control physical forces in a haphazard fashion, using only trial and error to obtain a lucky, hoped-for result, the process may be called "tinkering." When he attempts to order his manipulations into a more or less logical procedure, his efforts may be termed "engineering." The dividing line between tinkering and engineering is rarely sharp and distinct. On the one hand, it is virtually impossible for anyone but an idiot or an infant to manipulate physical things without doing some reasoning about it. On the other hand, important discoveries are sometimes accidentally made by random tinkering during the course of an engineering investigation. But, in general, we can say that manipulation of physical things can be called engineering only when the major portion of the effort involved is carried on in a logical planned manner, and where the amount of careful reasoning involved is restricted only by the law of diminishing returns.

The latter statement implies that a fair ability at reasoning is involved. The amount of useful thinking that a well-trained engineer can apply to a specific technical problem is far greater than the amount of reasoning that could be applied usefully by, say, a shepherd.

Granting, then, that a certain amount of reasoning is economically desirable in engineering work, we are faced with the problem of determining the quantity and quality of reasoning that will yield a maximum economic return in any given situation.

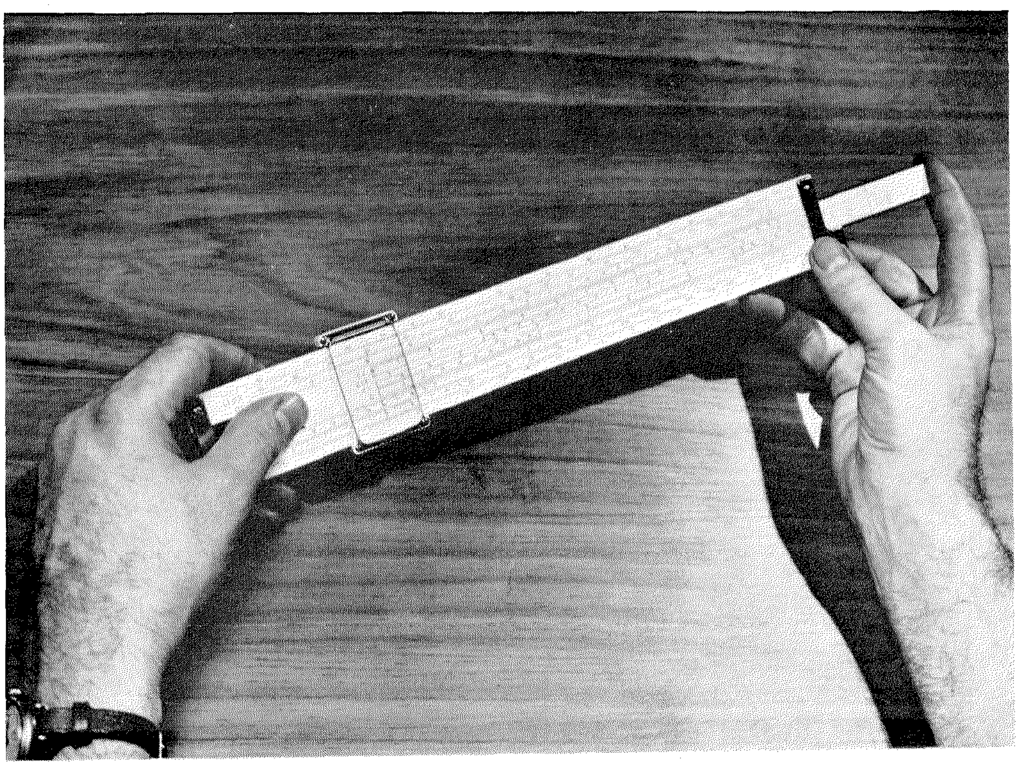
It is clear, of course, that this statement implies that there is at least a partial alternative to reasoning that at some point becomes more economical than reasoning itself. This alternative is, of course, experimentation.

The meaning of the term "quantity of reasoning" is fairly clear. It specifies how much reasoning shall be done.

The term "quality of reasoning" requires some discussion. It seems to imply that reasoning need not be precise to be acceptable. This, indeed, is one of the implications and its meaning will be explained shortly. But the term "quality" implies not only degree of precision, but degree of elaboration as well.

There are various kinds of situations in which it is fairly obvious that precise reasoning is unjustified, i.e., where there is no justification in attempting to predict exactly what will happen under certain circumstances. Broadly they may be classed as follows:

A. Situations where it doesn't matter whether or not you know exactly what is happening.



* Presented, August 15, 1949, before a group of engineers in training at Federal Telecommunication Laboratories, Incorporated, Nutley, New Jersey.

For example, in the bypass capacitor of a radio-frequency amplifier, it does not make any serious difference if the radio-frequency voltage across the capacitor is 1 percent of the plate voltage or 2 percent of the plate voltage, although these figures differ by 100 percent. As another example, as far as transmission of power is concerned, it usually doesn't matter if the characteristic impedance of a transmission line is known with an accuracy within no better than 10 percent.

In such cases, it is a waste of time to attempt to predict behavior within an accuracy of, say, 0.1 percent.

B. Situations where physical quantities are not known with sufficient accuracy to justify precise calculations based on them.

For example, in predicting the value of capacitance required to resonate an inductor at a specified frequency, there is no point in designing the capacitor within an error of 1 percent if the value of inductance is known only within 25 percent.

C. Situations in which the actual physical system under consideration is so complicated that quantitative reasoning about it is beyond the ability of any scientist.

In such cases, one "simplifies" the problem; i.e., one substitutes another problem that looks very much like the original problem, for which one expects for one reason or another the predicted behavior to be very much the same as the original system, and about which one can reason with some facility. For example, if the model shop prepares a hollow metallic object that is supposed to be a perfect right circular cylinder capped at the ends, but has the normal imperfections to be expected in machining, we acknowledge that it would be a waste of time to predict, exactly, the resonant frequencies of this cavity. Instead we simplify the problem by computing the resonant frequencies of the nominal cavity that the shop was supposed to try to build. We expect the actual frequencies to be pretty close to those predicted.

Sometimes, this process of substitution can be carried too far. Consider, for example, the operation of a class-C amplifier. When properly operated, the tube acts in a fashion similar to a resistive switch that opens and closes periodically. From a knowledge of the amount of drive applied to the grid of the tube, one can make a fair guess at a fixed value of resistance to be used in series with the switch, and proceed by conventional analytical means to predict the power output obtainable from such an amplifier. Thus, for the complex system of nonlinear tube characteristics associated with a resonant circuit we substitute a resonant circuit periodically shock-excited through a resistor.

This procedure might even be justified in attempting to predict the second-harmonic voltage produced across the tuned circuit. It would be completely out of line to attempt to predict the tenth harmonic, which varies strongly with the nonlinearities in the tube characteristics. An attempt to predict the tenth harmonic by this method would be a waste of time. An attempt to predict even the fifth harmonic by the more-direct attack on the tube characteristics would also be a waste of time since the tube characteristics are usually not known with the required accuracy.

The question of calculations that are too elaborate is a little more delicate. These are the calculations that obtain far more information than is needed, usually as a by-product of the process used in obtaining needed information.

To cite an extreme example, suppose we have a resonator made of a coaxial line capped at the ends. From measurements, we have determined that it resonates at a certain frequency. To be specific, let us suppose that it resonates at 500 megacycles per second. Suppose we ask, "How shall we change the dimensions of the cavity to make it resonate at 1000 megacycles?" The average engineer will tell us without hesitation to cut the length in half. The quick, almost instinctive, method of reasoning used here is the method of dimensional analysis, plus the knowledge that the resonant frequency is independent of cross-sectional shape. In external appearance, at least, it is the simplest form of reasoning that is sufficiently quantitative to be termed mathematical.

On the other hand, if the theory of resonant transmission lines were not known, one might be tempted to start with the engineering differential equations of the line, proceed to integrate them, insert boundary conditions, etc. Eventually, this process leads to the same conclusion as above, but with less economy of time, therefore money. True, additional knowledge has been gained as a result of the analysis. Voltage and current distributions are known. Power required to maintain a given degree of excitation is an easy by-product of the calculation. But this additional information, which was of no immediate use, has had to be paid for on a project that perhaps could ill afford it.

The situation might be even worse than this. One might be tempted to start with Maxwell's equations to obtain the desired result. Here the by-products are even more extensive. One obtains a picture of electric and magnetic field distributions in minute detail. Such information can be very useful—to some other project that needs the information and can afford to pay for it.

So much for the negative side of mathematics in engineering. What are the advantages of accurate analysis that should be stressed in the solution of engineering problems?

Frequently, situations arise in which there is no question in anybody's mind that the cost of appeal to experiment is, or could readily become, prohibitive. Some of these situations are introduced in the form of terse questions below.

Possible or Impossible?

Is it possible to design a network to transform an unbalanced input voltage to a balanced output voltage for any balanced load? Can such a network be a pi network? What are the minimum number of meshes such a network must have? If two sources are to excite a common load at the same frequency without reacting on each other, what must be the minimum insertion loss of the coupling network? Can an isotropic radiator be constructed? Can mass be converted into energy? These are some of the questions that are more readily answered, at least in part, by analysis rather than experiment.

In obtaining answers to questions of this sort, one must be very careful to interpret the results correctly. The analysis will state that results are possible or impossible *within the limitations imposed at the start of the analysis.* A result that is possible (or impossible) under certain conditions may be impossible (or possible) under others. The first law of thermodynamics states that it is possible to convert heat into mechanical energy. The second law of thermodynamics states that it is impossible to convert heat into mechanical energy without at the same time dumping heat into a cold reservoir. It is impossible to increase the bandwidth of an amplifier greatly without reducing its gain; but it is possible to increase the bandwidth without reducing the gain by adding enough amplifier stages,—provided you don't increase the bandwidth too much!

These ifs, ands, and buts of the possible and the impossible remind me of a little incident, which occurred to me when I was fresh out of college with a degree in mathematics and looking for a job. I met the chief engineer of one of our local radio stations. Like many engineers with not too much training in mathematics, he was overly impressed with the efficacy of this powerful tool in the hands of a tyro. But after a few sessions with me, in which we tried to find analytical solutions for some of his problems, he effected quite a rapid cure. Showing me to the door one day at the conclusion of an interesting, if somewhat fruitless, session he murmured with a sigh of regret, "We would make a wonderful team, I think, were it not for one thing. I'm afraid that if I worked with you for any length of time I would discover that most of the things I've been doing are impossible."

Many years later, I appreciated the importance of this remark. When a person constitutes himself as engineer and mathematician rolled into one, there is not often any serious problem of coordination of ideas between engineer and mathematician. If he asks a question and gets a negative answer, he backs up automatically to the starting point, finds what group of assumptions are leading to the negative result and, if the assumptions have been made purely for convenience (as they frequently are), he modifies them in order that his analysis may yield a more-favorable result. When, however, the functions of engineer and mathematician are separated into two highly specialized personalties, the utmost in patience and insight is frequently required on the part of both in order that they may function successfully as a team.

How?

It is not possible to ask mathematically "How shall I build a gadget to produce a certain result?" At least it is not possible to get an answer to such a broad question. But it is sometimes surprising how vaguely a question may be stated and yet lead to analysis that yields at least one constructive result. For example, within certain limitations one is permitted to ask, "How shall I construct a linear, passive, lumped-constant two-terminal pair network to yield a specified frequency response?" and expect to obtain a whole class of solutions satisfying his requirements. One may ask "Given a certain type of message and a certain type of interference added to it, what kind of filter shall I construct in order that its output may be as nearly like the message as possible?" One may ask this question and obtain a positive answer.

This process of starting an investigation by stating a desired result and from it evolving a design configuration is called synthesis. It is a more-advanced approach than straightforward analysis and is usually invoked only when previous experience fails to indicate a good starting point for analysis, or where the amount of trial and error involved might be less economical than direct synthesis.

How Much? How Many? How Big?

Probably the vast majority of problems fall in this category. Through past experience or through inventiveness, we become convinced (at least temporarily) that our design should take a certain qualitative form. We know the schematic diagram of a power supply, an audio-frequency amplifier, a Wheatstone bridge, but what shall be the values of the components to yield specified voltage, hum, transients, distortion, sensitivity, etc.? By the same token that most of the effort is applied in this field, the required design formulas exist to answer most of the questions. Here, care is required to see that formulas are not employed beyond the limitations within which they were derived.

Which One?

Sometimes our inventiveness is so great that we find ourselves faced with a choice of possible solutions. As a simple example, there is an infinite number of possible combinations of inductance and capacitance that will resonate at the same frequency. When this happens one or more additional restrictions must be imposed, and sometimes the restrictions are such that the solution can be found by analytical means. In production work, an additional restriction is frequently obtained by specifying that the total cost shall be a minimum. Frequently, in airborne designs, the additional restriction may be minimum weight or size. At other times, solutions must be nonmathematical, even nontechnical, and must involve such annoying considerations as ease of procurement.

Early in this paper it was indicated that the engineer is frequently faced with the problem of deciding how much analysis should be applied to a problem. The answer to this question usually involves, not merely the nature of the technical problem itself, but the nature of the analytical abilities at the disposal of the engineer, either within himself or among his associates. In general, the greater the available analytical talents, the more stress should be laid on analytical solutions.

But, —

If you are inclined to be analytical, don't be impatient with experimenters who prefer to get their answers in the laboratory. Ninety-nine times out of a hundred, they are choosing the method that will bring *them* the quickest results.

And if you are experimentally inclined, don't fall into one of two common errors. The first of these, which is more annoying but less dangerous, is to sneer at "pencil pushers." The more-dangerous not-at-all-annoying attitude reminds me of a facetious remark I heard from an engineer years ago to the effect that "nobody believes experimental data except the man who takes it, but everybody believes the results of a theoretical analysis except the man who makes it." The grain of truth in this remark stems from the awe with which theoretical work is viewed by some engineers. To get a result, analysts sometimes make assumptions that are not in accord with fact. If your intuition (born of experience) warns you that something is fishy about certain analytical conclusions, smoke the irritation out. You may be wrong or the analyst may be wrong; he may have committed a blunder or you may be unduly prejudiced by too-narrow experiences; but you have to meet each other on common ground and agree in your final conclusions.